⚙️ Phase 7 – Part 4: Scaling Laws & Limiting Behavior

## Core (reminder)

Plain text:  
Gravity(x) = (∇² [ space(x) + current(x)² ]) × ψ(x)

## 1. Goal (explicit)

I examine how the gravity field scales under spatial dilations and field rescalings, and I identify the dimensionless control ratios that determine whether the space term or the current² term dominates in different regimes. I then analyze limiting cases (current → 0, space → 0, ψ constant or sharply peaked) and derive practical diagnostics to decide dominance and asymptotic behavior.

## 2. General scaling transformation

Consider the scaling transformation of coordinates and fields:

Spatial rescale:

Field scalings: assume space(x), current(x), and ψ(x) scale as power laws under λ:

(Here α, β, γ are scaling exponents to be solved or chosen according to interpretation.)

Laplacian scales as:

Plain text:  
space(x) -> lambda^gamma \* space(lambda x)  
current(x) -> lambda^alpha \* current(lambda x)  
psi(x) -> lambda^beta \* psi(lambda x)  
∇² -> lambda^{-2} ∇²

Apply this to Gravity:

Plain text:  
Gravity\_lambda = lambda^{-2+gamma+beta} \* (∇² space)(lambda x)

* lambda^{-2+2alpha+beta} \* (∇² current²)(lambda x)

So the two contribution exponents are:

Interpretation: under spatial rescaling by λ, the space-contribution scales like λ^{E\_s} and the current-contribution like λ^{E\_c}. Which term dominates at large λ (coarse-graining / long wavelengths) depends on which exponent is larger.

## 3. Scale-invariance condition and special choices

Scale invariance of the entire Gravity field under (i.e., no net λ dependence) requires both exponents be zero:

Plain text:  
E\_s = -2 + gamma + beta = 0  
E\_c = -2 + 2\*alpha + beta = 0

From these two we get:

Plain text:  
gamma = 2 - beta  
alpha = 1 - beta/2

Thus, if I choose a ψ scaling exponent β, the required γ and α for scale invariance follow.

For example:

* If I set β = 0 (ψ dimensionless under scaling), then γ = 2 and α = 1. That is:

This is consistent with space having units of length² and current scaling like length (or velocity when time scaled similarly).

* If I choose β = 1, then γ = 1 and α = 1/2, etc.

I note these are bookkeeping relations — they help pick consistent physical units or renormalization group-like choices for model building.

## 4. Dimensionless control ratio (dominance criterion)

To decide which term dominates locally, define the dimensionless ratio:

Plain text:  
R(x) = current(x)² / space(x)

If R(x) ≫ 1, then current² dominates and curvature is set mainly by flow patterns.  
If R(x) ≪ 1, geometric space dominates and system reduces to space-driven curvature.

Because Laplacian acts on the sum, I also consider the relative magnitude of derivatives; a more refined local measure is:

Plain text:  
R\_tilde(x) = | ∇² (current(x)²) | / | ∇² (space(x)) |

## 5. Limiting cases (analytic simplifications)

**(A) current → 0 (pure space curvature)**

If current(x) ≡ 0 then:

Plain text:  
If current = 0: Gravity = (∇² space) \* psi

**(B) space → 0 (pure flow curvature)**

If space(x) ≡ 0 then:

Plain text:  
If space = 0: Gravity = (∇² current²) \* psi

**(C) ψ → constant (uniform modulation)**

If

then Gravity scales directly with the combined curvature:

Plain text:  
If psi = constant: Gravity = psi0 \* ∇²[space + current²]

**(D) ψ sharply localized (peaked)**

If ψ is sharply peaked at x₀ (e.g., Gaussian with small width), gravity is strongly localized near x₀, even if curvature contributions are broader. This is how I can produce compact gravity wells with modest curvature elsewhere.

**(E) Long-wavelength (coarse-grain) limit: λ → large**

Using exponents from section 2:

* If then at large scales the current-term grows faster and dominates.
* If then geometry dominates.

## 6. Concrete example (physical units and interpretation)

To ground the algebra I pick physically motivated units:

* Let space(x) have units of L² (e.g., effective geometric amplitude).
* Let current have units L/T (velocity) so current² has L²/T².

If I perform a pure spatial rescaling while keeping time fixed, typically current would scale like λ⁰ (if velocity is independent of scale) or like λ⁻¹ if I rescale time together. Because these choices depend on the physical setup (Eulerian vs Lagrangian viewpoint), I keep the general exponents α, γ, β and use the dimensionless ratio R(x) from §4 for diagnostics rather than a single universal rule.

Practical rule-of-thumb:

* For astronomical/slow flows, space term often dominates (R ≪ 1).
* For highly dynamic regions (accretion discs, shocks), current² can dominate (R ≫ 1).